Forthcoming: Econometrica

RETURNS TO TENURE OR SENIORITY? SUPPLEMENTARY MATERIAL: ADDITIONAL TABLES AND ESTIMATIONS *

I. Sebastian Buhai, Miguel A. Portela, Coen N. Teulings and Aico van Vuuren

We provide a web appendix to the paper "Returns to Tenure or Seniority?". It includes information on the residual autocovariances for within-job log wage innovations, in Appendix A, which we use to compute the standard deviation of the permanent shocks. In Appendix B, we show how one can estimate β_1 and β_2 for both the standard Topel and for the Topel variant with spell fixed effects specifications, when the model includes time dummy variables.

APPENDIX A: ADDITIONAL TABLES

TABLE VII

Residual autocovariances for within-job log wage innovations

Lag	Denmark 1980-2001	Portugal 1986-2009
0	0·0195	0·0355
	(0·00002)	(0·00007)
1	−0·0043	−0·0108
	(0·00001)	(0·00005)
2	−0·0004	−0·0009
	(0·00001)	(0·00002)
3	−0·0002	−0·0005
	(0·00001	(0·00003)
4	−0·0003	0·00001
	(0·00001)	(0·00003)
5	−0·00002	−0·00002
	(0·00001)	(0·00003)
6	−0·00009	−0·0005
	(0·00001)	(0·00003)
Number of observations	8,902,997	9,884,371

The generating regressions are the Topel regressions with seniority index included (see Table III in the paper). Only the first six lags are displayed here. Standard errors in parentheses.

An MA(1) process made up of a mixture of permanent and transitory shocks well describes the autocovariance patterns in Table VII. We perform a back-of-the-envelope computation of the standard deviation of the permanent shocks. Let q_{ijt} and u_{ijt} be the transitory and permanent shock, respectively. Then $\Delta v_{ijt} = u_{ijt} + q_{ijt} - q_{ij,t-1}$. Hence, Var($\Delta v_{ijt}$) =Var($u_{ijt}$) + 2Var($q_{ijt}$) and $\text{Cov}(\Delta v_{ijt}, \Delta v_{ij,t-1}) = -\text{Var}(q_{ijt})$, so that

$$Var\left(u_{ijt}\right) = Var\left(\Delta v_{ijt}\right) + 2Cov\left(\Delta v_{ijt}, \Delta v_{ij,t-1}\right).$$

We obtain 0.10 for Denmark and 0.12 for Portugal as standard deviation of the permanent shocks, which is in line with earlier estimates obtained for the US.

*Buhai: Stockholm University, sbuhai@gmail.com; Portela: University of Minho, mangelo@eeg.uminho.pt; Teulings: University of Amsterdam, c.n.teulings@uva.nl; van Vuuren: VU University Amsterdam, avuuren@feweb.vu.nl.

TABLE VIII

RETURNS TO JOB SENIORITY, TENURE, EXPERIENCE, AND FIRM SIZE, BASED ON VARIOUS REMAINING JOB DURATIONS IN FIRST-STEP MODEL (TABLE 4 OF TOPEL (1991), PANEL B)

	Remaining job durations in estimating wage growth					
	≥ 0	≥ 1	≥ 3	≥ 5	≥ 7	≥ 10
Denmark						
X_{ijt}	0·0478	0·0413	0·0341	0·0321	0·0332	0·0342
	(0·0004)	(0·0005)	(0·0006)	(0·0008)	(0·0010)	(0·0015)
T_{ijt}	−0·0082	−0·0067	−0·0027	−0·0054	−0·0058	−0·0124
	(0·0009)	(0·0010)	(0·0013)	(0·0016)	(0·0021)	(0·0030)
r_{ijt}	0·0079	0·0063	0·0027	0·0032	0·0019	0·0071
	(0·0003)	(0·0004)	(0·0005)	(0·0006)	(0·0008)	(0·0011)
n_{ijt}	0·0124	0·0181	0·0205	0·0231	0·0245	0·0227
	(0·0003)	(0·0004)	(0·0005)	(0·0006)	(0·0008)	(0·0010)
Portugal						
X_{ijt}	0·0690	0·0732	0·0767	0·0825	0·0819	0·0740
	(0·0000)	(0·0000)	(0·0000)	(0·0001)	(0·0001)	(0·0001)
T_{ijt}	0·0154	0·0146	0·0117	0·0092	0·0098	0·0115
	(0·0014)	(0·0016)	(0·0023)	(0·0033)	(0·0049)	(0·0078)
r_{ijt}	0·0150	0·0190	0·0221	0·0273	0·0262	0·0278
	(0·0004)	(0·0006)	(0·0008)	(0·0012)	(0·0016)	(0·0024)
n_{ijt}	0·0144	0·0126	0·0112	0·0108	0·0125	0·0145
	(0·0004)	(0·0006)	(0·0009)	(0·0012)	(0·0016)	(0·0024)

These are the same regressions as the Topel regressions with seniority index included from our main text (see corresponding column in Table III in the paper), but on various remaining job durations, replicating Topel (1991)'s Table 4, Panel B. Standard errors in parentheses.

APPENDIX B: ESTIMATION OF β_1 AND β_2 FOR STANDARD AND FE TOPEL, WHEN INCLUDING TIME DUMMY VARIABLES

B.1. Topel's model

Consider our empirical model as discussed in the main text of the paper

$$\log w_{ijt} = \beta_0 + \beta_{11} X_{ijt} + \beta_{12} X_{ijt}^2 + \beta_{21} T_{ijt} + \beta_{22} T_{ijt}^2 + \varepsilon_{ijt},$$

where we omit higher-order terms in X_{ijt} and T_{ijt}, as well as r_{ijt} and n_{jt}, for convenience. Taking within-spell first-differences, this results in

$$\begin{aligned}(11)\qquad \Delta \log w_{ijt} &= \beta_{11} + \beta_{21} + \beta_{12}\Delta X_{ijt}^2 + \beta_{22}\Delta T_{ijt}^2 + \Delta\tau_t + \Delta\nu_{ijt} \\ &= \beta_{11} + \beta_{21} + \tau_2 + \beta_{12}\Delta X_{ijt}^2 + \beta_{22}\Delta T_{ijt}^2 + \delta_t + \Delta\nu_{ijt},\end{aligned}$$

where we define for $t \geq 3$,

$$\delta_t = \Delta\tau_t - \tau_2,$$

or, by using repeated substitution, we obtain:

$$(12)\qquad \tau_t = (t-1)\tau_2 + \sum_{s=3}^{t} \delta_s.$$

Equation (11) shows that a regression of $\Delta \log w_{ijt}$ on ΔX_{ijt}^2, ΔT_{ijt}^2 and a full set of time dummies yields consistent estimates of $B = \beta_{11} + \beta_{21} + \tau_2$, β_{12} and β_{22}.

Substitution of (12) into (11), using $X_{ijt} = X_{ij,0} + T_{ijt}$ as well as $t = t_{ij,0} + T$, where $X_{0,ij}$ and $t_{ij,0}$ are respectively experience and time at the start of the spell, and replacing the coefficients by their estimates, we obtain:

$$\log w_{ijt} - \widehat{B}T_{ijt} - \widehat{\beta}_{12}X_{ijt}^2 - \widehat{\beta}_{22}T_{ijt}^2 - \sum_{s=3}^{t}\widehat{\delta}_s = \beta_0 + \beta_{11}X_{0,ij} + \tau_2(t_{0,ij} - 1) \tag{13}$$

This yields a second-stage regression on initial experience and year of job start to obtain β_{11} and τ_2.

B.2. *Topel with spell fixed effects*

Consider now a model in which we allow for heterogeneity in the linear returns to tenure

$$\log w_{ijt} = \beta_0 + \beta_{11}X_{ijt} + \beta_{12}X_{ijt}^2 + \beta_{21,ij}T_{ijt} + \beta_{22}T_{ijt}^2 + \varepsilon_{ijt}, \tag{14}$$

where we again omit higher-order terms for convenience. Note that this model is somewhat more restrictive than the model in the main text since it does not include individual heterogeneity in linear experience. Taking within-spell first-differences results in

$$\Delta\log w_{ijt} = \beta_{11} + \beta_{21,ij} + \tau_2 + \beta_{12}\Delta X_{ijt}^2 + \beta_{22}\Delta T_{ijt}^2 + \delta_t + \Delta\nu_{ijt} \tag{15}$$

where the definition of δ is the same as in Section B.1. Since we have that $\Delta X_{ijt}^2 = 2X_{ijt} - 1$ and $\Delta T_{ijt}^2 = 2T_{ijt} - 1$, this empirical model is also statistically equivalent to that of Altonji and Shakotko (1987), using within spell first-differences instead of the log wage level, and with $\beta_{21,ij} = \beta_2 + u_{\beta_2,ij}$,where $u_{\beta_{ij}}$ is a fixed job specific term. Hence, we can use Altonji and Shakotko's IV method to estimate equation (15), using $\widetilde{\Delta T_{ijt}^2} = 2\widetilde{T_{ijt}} = 2(T_{ijt} - \overline{T}_{ij})$ as an instrument for ΔT_{ijt}^2. As discussed in Topel (1991, p. 167), this method yields equivalent results to taking deviations from the mean combined with a second-step estimator to estimate β_{12} and β_{22}. Such a method yields estimates of $B_{ij} = \beta_{11} + \beta_{21,ij} + \tau_2$, β_{12} and β_{22}, as well as of the dummy variables δ_t. Using the same steps as in B.1. to obtain (13), we obtain:

$$\log w_{ijt} - \widehat{B}_{ij}T_{ijt} - \widehat{\beta}_{12}X_{ijt}^2 - \widehat{\beta}_{22}T_{ijt}^2 - \sum_{s=3}^{t}\widehat{\delta}_s = \beta_0 + \beta_{11}X_{0,ij} + \tau_2(t_{0,ij} - 1) \tag{16}$$

From this, β_{11} and τ_2 can be estimated. The estimate of $\beta_{21,ij}$ for every ij follows directly. It implies that we are able to identify the distribution of $\beta_{21,ij}$. We focus on the mean in the main text of our paper which can be consistently estimated by the sample average.

In order to calculate the standard errors, we can use the results of two-step estimators, taking into account that the first- and second- step error terms are correlated. More details are provided in, e.g., Murphy and Topel (1985, p. 94, equation 24) or Wooldridge (2002).

Assume now that instead of equation (14), we have a model that is equivalent with the model of our main text, hence allowing also for heterogenous effects in linear experience:

$$\log w_{ijt} = \beta_0 + \beta_{11,i} X_{ijt} + \beta_{12} X_{ijt}^2 + \beta_{21,ij} T_{ijt} + \beta_{22} T_{ijt}^2 + \varepsilon_{ijt},$$

Note that taking within spell first-differences yields a model that is statistically equivalent to the model in (15). The only difference is the interpretation of the predicted fixed effects that now equal $B_{ij} = \beta_{11,i} + \beta_{21,ij} + \tau_2$, where both $\beta_{11,i}$ and $\beta_{21,ij}$ include random components. Using the same techniques as that used to obtain (16) results in:

$$\log w_{ijt} - \widehat{B}_{ij} T_{ijt} - \widehat{\beta}_{12} X_{ijt}^2 - \widehat{\beta}_{22} T_{ijt}^2 - \sum_{s=3}^{t} \widehat{\delta}_s = \beta_0 + \beta_{11,i} X_{0,ij} + \tau_2 (t_{0,ij} - 1) \tag{17}$$

This equation is not feasible to estimate, since $\beta_{11,i}$ implies a dummy vector of a dimension that is as large as the number of individuals in the data set. In our paper (see Table III, and the returns to cumulated tenure in Table IV, both under the columns corresponding to the specification for Topel with spell fixed effects) we estimate therefore linear tenure and experience effects under the assumption of homogenous linear returns to experience, as illustrated above in equation (14). Of course, for identifying and estimating the object of our main interest in the paper, the return to seniority, we do not need any extra assumptions– see the main text, equation (10).

REFERENCES

Altonji, J.G. and R.A. Shakotko (1987). "Do wages rise with seniority?". *Review of Economic Studies*, **54**, 437–459.

Murphy, Kevin M. and Robert H. Topel (1985). "Estimation and Inference in Two-Step Econometric Models". *Journal of Business & Economic Statistics*, **3**, 370–379.

Topel, R.H. (1991). "Specific capital, mobility and wages: wages rise with job seniority". *Journal of Labor Economics*, **99**, 145–175.

Wooldridge, J.M. (2002). *Econometric Analysis of Cross Section and Panel Data*, MIT Press, Cambridge (MA).